

There are 4 different ways that we may write exponential functions depending on the information given:

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- 1<sup>st</sup> Way:

$$A = a(1+r)^t \text{ where:}$$

A = the amount after time t

a = the initial amount

r = **the percentage growth rate**

t = time in years, months, days, hours, etc...

- We use this formula when the problem says something like ....

“A population of size 200 is growing at 3.2% annually”

- 2<sup>nd</sup> Way:

$$A = a(1-r)^t$$

A = the amount after time t

a = the initial amount

r = **the percentage growth rate**

t = time in years, months, days, hours, etc...

- We use this formula when the problem says something like ....

“A population of size 200 is decreasing by 3.2% annually”

- 3<sup>rd</sup> Way:

$$A = a(b)^{t/n} \text{ where:}$$

A = the amount after time t

b = the population multiplier or **growth factor**

t = time

n = the amount of time needed for the growth factor to apply to the population

- We use this formula when the problem says something like ....  
“A population of size 10 triples every 5 days”

- 4<sup>th</sup> Way:

$$A = old\left(\frac{new}{old}\right)^{t/n} \text{ where:}$$

A = the amount after time t

old = the initial amount

new = a second later amount

t = time

n = the time it takes for the population to grow or decline from the old amount to the new amount

- We use this formula when the problem says something like ....  
“A population of size 250 grew to a size of 400 after a 3 year period”

\*\* Notice that all of these 4 forms of our exponential function look very similar:

$$A = \underset{(a)}{\text{(some initial value)}} \times \underset{(1+r \text{ or } b \text{ or new/old})}{\text{(some multiplier)}} \text{ raised to either } t \text{ or } t/n$$

This begs the question of whether or not we can convert from one form to the other. That is if we are given (or can find) the population multiplier (b) and how long it takes for (b) to apply can we determine the growth rate r?

Example 1:

In 1983 a population had size 120 and grew to a size of 165 by 1984. Find the annual growth rate for this population.

Example 2:

A decomposing material has a size of 40grams at noon but has declined to a size of 24 grams by 3PM. Find the hourly percentage decomposition rate for this material.

1. Write the exponential function that models the following situations:

a. A population of size 20 is decreasing by 0.5% every year.

b. A population of size 350 is growing 6 fold every 25 years.

c. In 1990 a town had a population of 1200 and it grew to a size of 1500 by 2003.

2. Using your function from part c above predict the following:

a. What will the town's population be in 2008?

b. In what month of what year will the town's population reach 4000 people?

3. Suppose that \$850 is deposited into an account paying 2.8% annual interest.

a. How much money will be in the account after 10 years?

b. How many years will it take for the money in the account to triple?

4. Steve has \$2500 to invest and would like to have his money to grow to \$10000 after just 6 years. What annual interest rate would make this possible?

5. A naturally decaying element population measures 120 grams. After 45 days the element has reduced in size to 89 grams.

a. Write an exponential model that gives the size of the element after  $t$  days.

b. Predict the size of the element after 1 year.

c. After how many days will the element be at  $\frac{1}{4}$  its original size?

d. What is the daily growth rate for the element?

6. A cake is removed from a 400 degree oven at noon and placed on a table where the room temperature is 71 degrees. The table below gives the actual temperature of the cake for several times after being removed from the oven. Find the time on the clock when the cake will reach a temperature of 95 degrees?

|      |       |       |       |       |      |
|------|-------|-------|-------|-------|------|
| time | 12:00 | 12:15 | 12:30 | 12:45 | 1:00 |
| temp | 400   | 340   | 265   | 220   | 195  |

7. Use the table below to answer the question that follow.

|                         |     |     |     |     |     |
|-------------------------|-----|-----|-----|-----|-----|
| Years after 1960        | 0   | 10  | 15  | 20  | 25  |
| population in thousands | 120 | 132 | 150 | 240 | 370 |

- a. Find the best fit exponential model for the population of the town.
- b. Find the SSE and average error for your model in part a.
- c. Predict the population in the year 2000.
- d. Predict when the population will reach 1 million.
- e. Based on your model what is the annual growth rate for this population?

8. Your salary is currently \$12,000 per month of which 16% goes towards taxes. Next year you will receive a 20% raise. Every dollar that you earn above \$12,000 will be taxed at a marginal rate of 32%. What is the percentage increase in your take home pay?